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Fermilab
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Beam Steering in the Transition Section

of the Fermilab Linac Upgrade

Fermilab Report - Linac Upgrade Note 216

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I. Overview:

In a previous Linac Upgrade Note[1], beam steering through the accelerating region of the Linac Upgrade was examined. This note will consider the correction of beam position and angle errors at the exit of the Drift-Tube Linac (DTL) through the use of three dipole magnets in the transition section of the Linac Upgrade (LU).

II. Theory:

The lattice of the transition section is shown schematically in Figure #1. The beam position x_0 is measured by the BPM at the exit of DTL Tank 5. The lattice is composed of three dipole magnets and three quadrupole magnets as well as drift spaces. The quadrupole magnets are modeled as thin lens quadrupoles with a focal length of f_i (for the ith quadrupole). For the design value of quadrupole strength, the thin lens approximation is valid to within 5%. The dipole magnets are characterized by the angular kick d_i imparted to the beam. Between components are drift spaces of length l_i. From these parameters, the beam position at Dipole #3 (x, x') can be calculated in terms of the initial position at the Tank 5 BPM (x_0, x_0'). This calculation quickly becomes a mammoth exercise in matrix algebra and further details are reserved for Appendix I. We desire the beam position and angle to be zero at Dipole #3 and want to determine the appropriate values of the dipole magnets d_i. The final result can be thought of as:

$$x = Q\,x_o + R\,x_o' + T\,d_1 + S\,d_2 = 0 \tag{1}$$

$$x' = U\,x_o + V\,x_o' + W\,d_1 + Y\,d_2 + d_3 = 0 \tag{2}$$

where Q, R, T, S, U, V, W, and Y, are simply functions of known values i.e. drift spaces l_i and quadrupole strengths f_i. The beam position x_o is measured however the angle x_o' is not, therefore we have 2 equations with 4 unknowns. Although the value of the angle x_o' is not known, it is a constant. Thus we can envision equations 1 and 2 as providing us with d_2 and d_3 as functions of d_1 and can construct:

$$S = d_1^2 + d_2^2 + d_3^2 \tag{3}$$

and require S to be a minimum by:

$$\frac{\partial S}{\partial d_1} = 2d_1 + 2d_2\frac{\partial d_2}{\partial d_1} + 2d_3\frac{\partial d_3}{\partial d_1} = 0 \tag{4}$$

which from equations 1 and 2 becomes:

$$d_1 + d_2\left(\frac{-T}{S}\right) + d_3\left(\frac{TY}{S} - W\right) = 0 \tag{5}$$

From equation 1, 2, and 5 we solve for the three unknowns d_2, d_3, and x_o':

$$d_2 = \frac{x_o\left(\frac{TY}{S} - W\right)\left(\frac{QV}{R} - U\right) + d_1\left\{1 + \left(\frac{TY}{S} - W\right)\left(\frac{TV}{R} - W\right)\right\}}{\frac{T}{S} + \left(\frac{TY}{S} - W\right)\left(Y - \frac{VS}{R}\right)} \tag{6}$$

$$d_3 = \frac{d_1 - \frac{T}{S}d_2}{\left(W - \frac{TY}{S}\right)} \tag{7}$$

$$x_o' = \frac{-1}{R}\left[Qx_o + Td_1 + Sd_2\right] \tag{8}$$

To summarize then, from a measured value of x_o we vary d_1 and have d_2 and d_3 follow according to equations 6 and 7. When the value of d_1 is set correctly then equation 8 is satisfied and the beam will be correctly steered at dipole #3. This will be observable by monitoring the BPM located in the quadrupole preceding the first accelerating module.

III. Calculations:

The following table provides the basic data used in these calculations. The drift space lengths are taken from design drawings (#0230.0000 ME-62910 1 and 2 of 16) of the transition section. Quadrupole strengths are design values[2].

Drift Space	Length (cm)	Quadrupole	1/f (1/cm)
L1	20.8	#1 (H)	-0.0113
L2	34.6	#2 (V)	0.00718
L3	154.0	#3 (H)	-0.0111
L4	14.8		
L5	88.6		
L6	14.9		

From these values we obtain:

Q = -2.31	U = 0.0108 (radians/cm)
R = 214 (cm)	V = -1.43
T = 262 (cm)	W = -1.65
S = 88.9 (cm)	Y = 0.0158

and thus:

$$d_2 = (0.00265)x_o + (0.362)d_1$$

$$d_3 = (0.0046)x_o + (0.0394)d_1$$

where x_o is in cm and d_1 d_2 d_3 are in radians.

These equations were set up within a Microsoft Excel spreadsheet (Figure #2) and used to determine the range of initial conditions over which the three dipoles would be able to correct the beam position and angle errors. The original design specifications of the dipole magnets called for the production of 1.5 milliradian kicks[3]. Subsequent testing of the magnets shows a maximum angular displacement of 2.5, 3.1, and 3.1 milliradians for the three dipoles in the transition section[4]. The first dipole has a smaller displacement due to fewer windings to enable the magnet to fit the available space. The results of these calculations are shown in Figure #3. For a given beam position error, a window of 6.6 milliradians is available in which the three-bump of the dipole magnets will be able to correct the beam position and angle. In addition to the capabilities of the system considered, existing dipoles at the entrance to DTL Tanks 3, 4, and 5 could also be utilized to steer the beam prior to the three-bump.

IV. References:

[1]K.L. Junck, "Beam Steering in the Fermilab Linac Upgrade", Fermilab Report - Linac Upgrade Note #211, May 1993.

[2]J.A. MacLachlan, "Transition Section Design Rationale and New Parameters", Fermilab Report - Linac Upgrade Note #158, April 1990.

[3]"Fermilab Linac Upgrade Conceptual Design", Revision 4A, November 1989.

[4]E.S. McCrory, "Beam Diagnostics for the Fermilab 400 MeV Linac Upgrade", Fermilab Report - Linac Upgrade Note, May 1993.

Schematic of the Transition Section of the Linac Upgrade

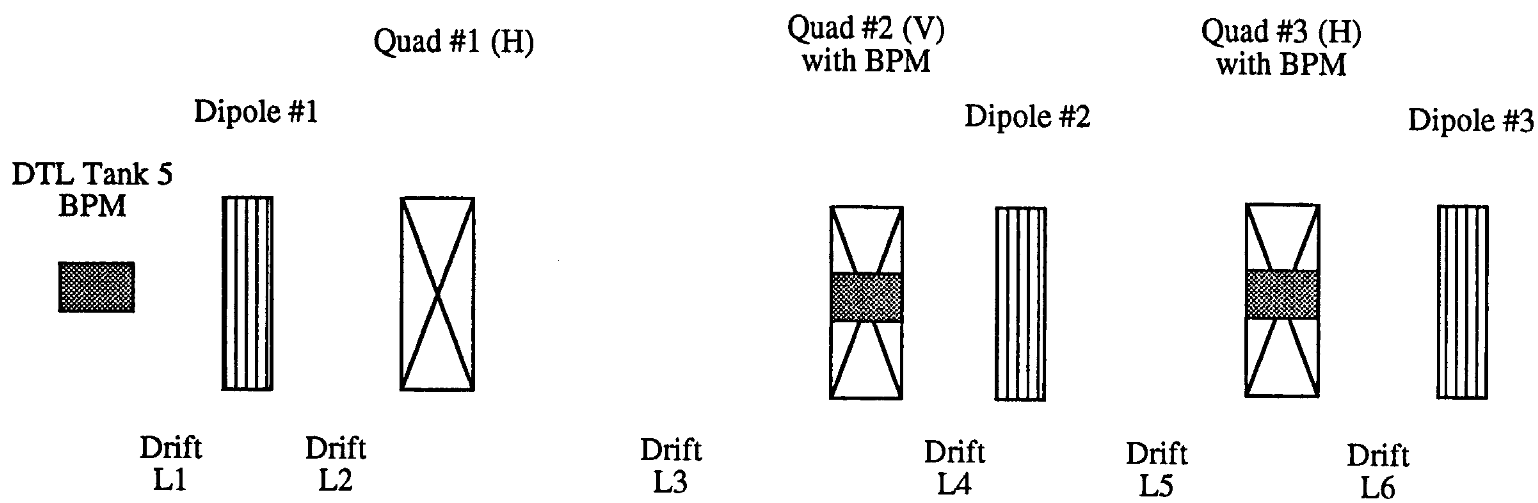

Figure #1

transition.steering

Device	L (cm)	D (mr)	D (rad)	Sqrt(K)*l	1/f (1/cm)	X (cm)	X' (rad)
BPM5OT						0.4	1.64E-03
Drift (l1)	20.8					0.4	1.64E-03
Dipole #1		1.800	1.80E-03			0.4	3.44E-03
Drift (l2)	34.6					0.6	3.44E-03
Quad #1 (H)				0.311	-1.13E-02	0.6	-2.83E-03
Drift (l3)	154.0					0.1	-2.83E-03
Quad #2 (V)				0.247	7.18E-03	0.1	-1.99E-03
Drift (l4)	14.8					0.1	-1.99E-03
Dipole #2		1.176	1.18E-03			0.1	-8.14E-04
Drift (l5)	88.6					0.0	-8.14E-04
Quad #3 (H)				0.308	-1.11E-02	0.0	-9.86E-04
Drift (l6)	14.9					0.0	-9.86E-04
Dipole #3		0.981	9.81E-04			0.0	-5.29E-06
Drift	53.7					0.0	-5.29E-06
kappa =	8.35E-01			lambda=	-1.22E-02	sum dip=	5.59
m =	1.56E+00			rho=	-2.02E+00	x0'=	1.64
n=	3.42E+02			psi=	1.58E-02		
s=	8.89E+01			u=	1.08E-02		
q=	-2.31E+00			v=	-1.43E+00		
r=	2.14E+02			w=	-1.65E+00		
t=	2.62E+02						

Figure #2

Phase Space at Tank 5 Output BPM for which
Transition Section 3-bump can correct

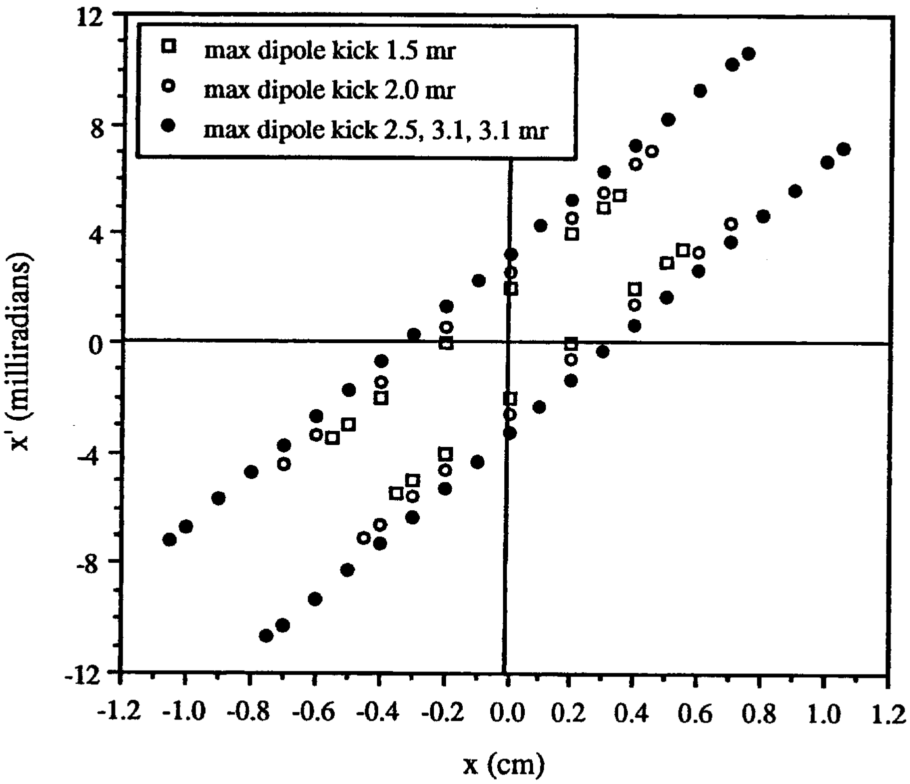

Figure #3

T5 BPM OUT $\quad X_0, \; X_0'$

drift ℓ_1 $\quad X_0 + \ell_1 X_0', \; X_0'$

dipole d_1 $\quad X_0 + \ell_1 X_0', \; X_0' + d_1$

drift ℓ_2 $\quad X_0 + \ell_1 X_0' + \ell_2 (X_0' + d_1), \; X_0' + d_1$

quad #1 $\begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix}$ $\quad \underbrace{X_0 + \ell_1 X_0' + \ell_2 (X_0' + d_1)}_{\equiv \alpha} \; ; \; \underbrace{\frac{-1}{f_1}\left(X_0 + \ell_1 X_0' + \ell_2 (X_0' + d_1)\right) + X_0' + d_1}_{\beta}$

drift ℓ_3 $\quad x = \alpha + \ell_3 \beta$

$x' = \beta$

quad #2 $\begin{bmatrix} 1 & 0 \\ 1/f_2 & 1 \end{bmatrix}$ $\quad x = \alpha + \ell_3 \beta$

$x' = \frac{1}{f_2}(\alpha + \ell_3 \beta) + \beta$

drift ℓ_4 $\quad x = \alpha + \ell_3 \beta + \ell_4 \left(\frac{1}{f_2}(\alpha + \ell_3 \beta) + \beta \right)$

$x' = \frac{1}{f_2}(\alpha + \ell_3 \beta) + \beta$

dipole d_2 $\quad x = \alpha + \ell_3 \beta + \ell_4 \beta + \ell_4 \frac{1}{f_2}(\alpha + \ell_3 \beta)$

$x' = \frac{1}{f_2}(\alpha + \ell_3 \beta) + \beta + d_2$

drift ℓ_5 $\quad x = \alpha + \ell_3 \beta + \ell_4 \beta + \ell_4 \frac{1}{f_2}(\alpha + \ell_3 \beta) + \ell_5 \left[\frac{1}{f_2}(\alpha + \ell_3 \beta) + \beta + d_2 \right] = \gamma$

$x' = \frac{1}{f_2}(\alpha + \ell_3 \beta) + \beta + d_2 \quad = \varepsilon$

quad #3 $\begin{bmatrix} 1 & 0 \\ -1/f_3 & 1 \end{bmatrix}$ $\quad x = \gamma \quad x' = \frac{-1}{f_3}\gamma + \varepsilon$

drift ℓ_6 $\quad x = \gamma + \ell_6 \left(\frac{-1}{f_3}\gamma + \varepsilon \right) \quad x' = \frac{-1}{f_3}\gamma + \varepsilon$

dipole d_3 $\quad \boxed{x = \gamma + \ell_6 \left(\frac{-1}{f_3}\gamma + \varepsilon \right) \quad x' = \frac{-1}{f_3}\gamma + \varepsilon + d_3}$

(2) $$x' = \frac{-1}{f_3}\gamma + \frac{1}{f_2}(\alpha + l_3\beta) + \beta + d_2 + d_3$$

$$x' = \frac{-1}{f_3}\left\{\alpha + l_3\beta + l_4\beta + l_4\frac{1}{f_2}(\alpha + l_3\beta) + l_5\left[\frac{1}{f_2}(\alpha + l_3\beta) + \beta + d_2\right]\right\} + \frac{1}{f_2}(\alpha + l_3\beta) + \beta + d_2 + d_3$$

$$x' = \frac{-1}{f_3}\alpha + \frac{-1}{f_3}l_3\beta + \frac{-1}{f_3}l_4\beta + \frac{-1}{f_3}\frac{1}{f_2}l_4\alpha + \frac{-1}{f_3}\frac{1}{f_2}l_4 l_3\beta$$
$$+ \frac{-1}{f_3}l_5\frac{1}{f_2}\alpha + \frac{-1}{f_3}l_5\frac{1}{f_2}l_3\beta + \frac{-1}{f_3}l_5\beta + \frac{-1}{f_3}l_5 d_2 + \frac{1}{f_2}\alpha + \frac{1}{f_2}l_3\beta + \beta + d_2 + d_3$$

$$x' = \alpha\left[\frac{-1}{f_3} + \frac{-1}{f_3}\frac{1}{f_2}l_4 + \frac{-1}{f_3}\frac{1}{f_2}l_5 + \frac{1}{f_2}\right]$$
$$+ \beta\left[\frac{-1}{f_3}l_3 + \frac{-1}{f_3}l_4 + \frac{-1}{f_3}\frac{1}{f_2}l_4 l_3 + \frac{-1}{f_3}\frac{1}{f_2}l_3 l_5 + \frac{-1}{f_3}l_5 + \frac{1}{f_2}l_3\right]$$
$$+ \frac{-1}{f_3}l_5 d_2 + \beta + d_2 + d_3$$

$$x = \gamma + l_6\left(\frac{-1}{f_3}\gamma + \varepsilon\right)$$

$$x = \left[\alpha + l_3\beta + l_4\beta + l_4\frac{1}{f_2}\alpha + l_4\frac{1}{f_2}l_3\beta + l_5\beta + l_5 d_2 + l_5\frac{1}{f_2}\alpha + l_5\frac{1}{f_2}l_3\beta\right]\underbrace{\left(1 + l_6\frac{-1}{f_3}\right)}_{\equiv K}$$
$$+ l_6\frac{1}{f_2}\alpha + l_6\frac{1}{f_2}l_3\beta + l_6\beta + l_6 d_2$$

$$x = \alpha\left[K + l_4\frac{1}{f_2}K + K l_5\frac{1}{f_2} + l_6\frac{1}{f_2}\right]$$
$$+ \beta\left[l_3 K + l_4 K + K l_4\frac{1}{f_2}l_3 + K l_5 + K l_5\frac{1}{f_2}l_3 + l_6\frac{1}{f_2}l_3 + l_6\right]$$
$$+ K l_5 d_2 + l_6 d_2$$

$$x = \alpha\left\{K\left(1 + \frac{1}{f_2}(l_4 + l_5)\right) + \frac{1}{f_2}l_6\right\}$$
$$+ \beta\left\{K\left(l_3 + l_4 + \frac{1}{f_2}l_3 l_4 + l_5 + \frac{1}{f_2}l_3 l_5\right) + l_6\left(1 + \frac{1}{f_2}l_3\right)\right\}$$
$$+ d_2\left(K l_5 + l_6\right)$$

$$X = \alpha \left\{ X\left[1 + \frac{1}{f_2}(l_4 + l_5)\right] + \frac{1}{f_2} l_6 \right\}$$

$$+ \beta \left\{ K l_3 + (1 + {}^1\!/\!_{f_2} l_3)\left[X(l_4 + l_5) + l_6\right] \right\}$$

$$+ d_2 \underbrace{(K l_5 + l_6)}_{S}$$

$$X' = \alpha \left[\frac{-1}{f_3}\left(1 + \frac{1}{f_2} l_4\right) + \frac{1}{f_2}\left(1 + \frac{-1}{f_3} l_5\right) \right]$$

$$+ \beta \left[1 + \frac{-1}{f_3}(l_3 + l_4 + l_5) + \frac{1}{f_2} l_3 \left(1 + \frac{-1}{f_3}(l_4 + l_5)\right) \right]$$

$$+ d_2 \underbrace{\left(1 + \frac{-1}{f_3} l_5\right)}_{\psi}$$

$$+ d_3$$

$X = m\alpha + n\beta + s d_2$ $\qquad$ m, n, s functions of f_i, l_i

$$X = m(X_0 + l_1 X_0' + l_2 X_0' + l_2 d_1) + n\left(\frac{-1}{f_1}(X_0 + l_1 X_0' + l_2 X_0' + l_2 d_1) + X_0' + d_1\right) + s d_2$$

$$X = X_0 \overbrace{\left\{ m + n\frac{-1}{f_1} \right\}}^{Q} + X_0' \overbrace{\left\{ m(l_1 + l_2) + n\frac{-1}{f_1}(l_1 + l_2) + n \right\}}^{R}$$

$$+ d_1 \underbrace{\left[m l_2 + n\left(1 + \frac{-1}{f_1} l_2\right) \right]}_{T} + S d_2$$

$$d_2 = \frac{-T}{S} d_1 + \frac{-Q}{S} X_0 + \frac{-R}{S} X_0'$$

$X = q X_0 + r X_0' + T d_1 + S d_2$ $\qquad$ q, r, s, T fns of f_i, l_i

$$X' = \lambda\alpha + \rho\beta + \psi d_2 + d_3$$

$$X' = \lambda(X_0 + l_1 X_0' + l_2(X_0' + d_1)) + \rho\left[\frac{-1}{f_1}(X_0 + l_1 X_0' + l_2(X_0' + d_1)) + X_0' + d_1\right] + \psi d_2 + d_3$$

$$= \lambda X_0 + \lambda(l_1 + l_2) X_0' + \lambda l_2 d_1 + \rho\frac{-1}{f_1} X_0 + \rho\left(\frac{-1}{f_1}(l_2 + l_1) + 1\right) X_0' + d_1(\rho)\left(1 + \frac{-1}{f_1} l_2\right) + \psi d_2 + d_3$$

$$= X_0 \left\{ \lambda + \rho\frac{-1}{f_1} \right\} + X_0' \left\{ \lambda(l_1 + l_2) + \rho\left[1 + \frac{-1}{f_1}(l_1 + l_2)\right] \right\}$$

$$+ d_1 \left\{ \lambda l_2 + \rho\left(1 + \frac{-1}{f_1} l_2\right) \right\} + \psi d_2 + d_3$$

$$d_3 = \left(\frac{T\psi}{S} - W\right) d_1 + \left(\frac{Q\psi}{S} - U\right) X_0 + \left(\frac{R\psi}{S} - V\right) X_0'$$

$$X' = U X_0 + V X_0' + W d_1 + \psi d_2 + d_3$$

$(1) \rightarrow \quad X_0 = \frac{-1}{R}\left[Q X_0 + T d_1 + S d_2 \right]$

$(2) \quad d_3 = -Y d_2 - W d_1 - U X_0 + \frac{V}{R}\left[Q X_0 + T d_1 + S d_2 \right]$

$$d_3 = X_0\left(\frac{QV}{R} - U\right) + d_2\left(\frac{VS}{R} - Y\right) + d_1\left(\frac{TV}{R} - W\right)$$

$$S = d_1^2 + d_2^2 + d_3^2 \qquad \frac{\partial S}{\partial d_1} = 2d_1 + 2d_2\frac{\partial d_2}{\partial d_1} + 2d_3\frac{\partial d_3}{\partial d_1} = 0$$

$$d_1 + d_2\left(-T/S\right) + d_3\left(\frac{TY}{S} - W\right) = 0$$

$$d_1 + d_2\left(-T/S\right) + \left(\frac{TY}{S} - W\right)\left(\frac{QV}{R} - U\right)X_0 + \left(\frac{TY}{S} - W\right)\left(\frac{VS}{R} - Y\right)d_2 + \left(\frac{TY}{S} - W\right)\left(\frac{TV}{R} - W\right)d_1 = 0$$

$$d_2\left[\frac{+T}{S} - \left(\frac{TY}{S} - W\right)\left(\frac{VS}{R} - Y\right)\right] = X_0\left[\left(\frac{TY}{S} - W\right)\left(\frac{QV}{R} - U\right)\right] + d_1\left[1 + \left(\frac{TY}{S} - W\right)\left(\frac{TV}{R} - W\right)\right]$$

$$1 + \frac{T^2Y^2}{S^2} + W^2 - 2\frac{TY}{S}W + \left(\frac{-TY}{S} + \frac{TV}{R}\right)\left(\frac{TY}{S} - W\right)$$

$$\left(\cancel{\frac{-T^2Y^2}{S^2}} + \frac{TYW}{S} + \frac{TVTY}{RS} - \frac{WTV}{R}\right.$$

$$1 + W^2 - \frac{TY}{S}W + \frac{TVTY}{RS} - \frac{WTV}{R}$$

$$1 + \frac{TYTV}{RS} - \frac{WTY}{S} - \frac{WTV}{R} + W^2$$